AP® CALCULUS BC
2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°\text{F}$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$
$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately $1.017\ °\text{F}$ per minute at time $t = 12$ minutes.

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by $16\ °\text{F}$ over the interval from $t = 0$ to $t = 20$ minutes.

$2: \begin{cases} 1: \text{value} \\ 1: \text{interpretation with units} \end{cases}$

(c)
$$\begin{aligned}\frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ &= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ &= \frac{1}{20} \cdot 1215.8 = 60.79\end{aligned}$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{approximation} \\ 1: \text{underestimate with reason} \end{cases}$

(d)
$$\begin{aligned}W(25) &= 71.0 + \int_{20}^{25} W'(t)\,dt \\ &= 71.0 + 2.043155 = 73.043\end{aligned}$$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{67.9-61.8}{15-9} = \boxed{1.0167 \text{ °F/min}}$$

At $t=12$, the temperature of the water in the tub is increasing at the rate of 1.067 °F/min.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = \boxed{16\text{°F}}$$

$\int_0^{20} W'(t)\,dt$ is the difference in temperature in °F of the water in the tub at $t = 20$ and $t = 0$.

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Continue problem 1 on page 5.

1A4

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\int_0^{20} W(t)\,dt \approx 4(55.0) + 5(57.1) + 6(61.8) + 5(67.9)$$
$$= 1215.8$$
$$\frac{1}{20}\int_0^{20} W(t)\,dt = \boxed{60.79\,°F}$$

As the function $W(t)$ is strictly increasing, the approximation rectangles of the left Riemann sum fall below the curve. Thus the approximation is an $\boxed{\text{underestimate.}}$

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$\int_{20}^{25} W'(t) = W(25) - W(20) = 2.043$$
$$W(25) - 71.0 = 2.043$$
$$W(25) = \boxed{73.0432\ °F}$$

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t (minutes)	0	4	9	15	20
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1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) = \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = \frac{6.1}{6} = 1.016 \frac{°F}{min}$$

At time $t = 12$, the water is being heated at a rate of $1.016 \frac{°F}{min.}$

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(20) - W(0)$$

$$= 71.0 - 55.0 = 16.0 \text{ degrees Fahrenheit}$$

$\int_0^{20} W'(t)\,dt$ represents the change in water temperature in degrees Fahrenheit from time $t=0$ to $t=20$.

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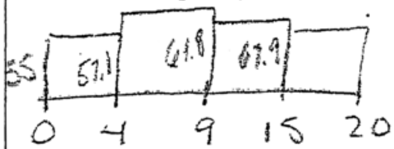
Continue problem 1 on page 5.

1B₂

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



$$\frac{1}{20}\left[55(4-0)+57.1(9-4)+61.8(15-9)+67.9(20-15)\right]$$

$$\frac{1}{20}\left[55(4)+57.1(5)+61.8(6)+67.9(5)\right]$$

$$\frac{1}{20}\left[1215.8\right] = 60.79$$

This approximation underestimates the average temp over the 20 minutes because $W''(t) > 0$. Concave up underestimates

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$W'(25) = 0.4\sqrt{25}\cos(.06(25))$$
$$= 0.141$$

$$y - 71.0 = 0.141(x - 20)$$
$$y = 0.141x - 2.82 + 71$$
$$y = 0.141x + 68.18$$

$$w(25) = 0.141(25) + 68.5$$
$$= 71.705$$

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 1C1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{W(t)_2 - W(t)_1}{t_2 - t_1}$$

$$\approx \frac{W(9) - W(4)}{9-4}$$

$$\approx \frac{61.8 - 57.1}{9-4} \approx \frac{4.7}{5} \boxed{\approx .94 \text{ degrees/min}}$$

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)dt = W(t)\Big]_0^{20}$$

$$W(20) - W(0)$$

$$71 - 55 = \boxed{16 \text{ degrees Fahrenheit}}$$

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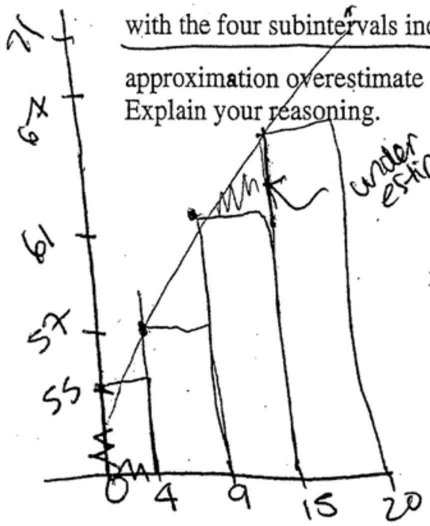
1C2

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\boxed{\frac{1}{20}\int_0^{20} W(t)\,dt}$. Use a $\boxed{\text{left}}$ Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



$(4-0)(55)+(9-4)(57.1)+(15-9)(61.8)$
$+ (20-15)(67.9)$

$220 + 285.5 + 370.8 + 339.5$

$\frac{1215.8}{20} \approx$ 60.79 avg temp

★ ~~This~~ This approximation underestimates the average temperature as seen in the graph above. Since the height of the rectangles are being cut short, the estimate is smaller than the actual.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

20.43154699

20°F

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Question 1

Overview

This problem involved a function W that models the temperature, in degrees Fahrenheit, of water in a tub. Values of $W(t)$ at selected times between $t = 0$ and $t = 20$ minutes are given in a table. Part (a) asked students for an approximation to the derivative of the function W at time $t = 12$ and for an interpretation of the answer. Students should have recognized this derivative as the rate at which the temperature of the water in the tub is increasing at time $t = 12,$ in degrees Fahrenheit per minute. Because $t = 12$ falls between the values presented in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 12$ that is supported by the table. Part (b) asked students to evaluate the definite integral $\int_0^{20} W'(t)\,dt$ and to interpret the meaning of this definite integral. Students should have applied the Fundamental Theorem of Calculus and used values from the table to compute $W(20) - W(0).$ Students should have recognized this as the total change in the temperature of the water, in degrees Fahrenheit, over the 20-minute time interval. In part (c) students were given the expression for computing the average temperature of the water over the 20-minute time period and were asked to use a left Riemann sum with the four intervals given by the table to obtain a numerical approximation for this value. Students were asked whether this approximation overestimates or underestimates the actual average temperature. Students should have recognized that for a strictly increasing function, the left Riemann sum will underestimate the true value of a definite integral. In part (d) students were given the symbolic first derivative $W'(t)$ of the function W that models the temperature of the water over the interval $20 \le t \le 25,$ and were asked to use this expression to determine the temperature of the water at time $t = 25.$ This temperature is computed using the expression $W(25) = W(20) + \int_{20}^{25} W'(t)\,dt,$ where $W(20) = 71$ is given in the table.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and no points in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student earned the left Riemann sum and approximation points. The student does not give a correct reason for "underestimates," so the last point in part (c) was not earned. In part (d) the student's work is incorrect.

Sample: 1C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In parts (a) and (d) the student's work is incorrect. In part (b) the student earned the value point. In part (c) the student earned the left Riemann sum and approximation points. The student does not give a correct reason for "underestimates," so the last point in part (c) was not earned.

Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\dfrac{dx}{dt}\right|_{t=2} = \dfrac{2}{e^2}$

Because $\left.\dfrac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055$ (or 3.054)

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \dfrac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \displaystyle\int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \displaystyle\int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$
$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2A₁

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

x y

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$

@ $t=2$,

$\frac{dx}{dt} = \frac{\sqrt{2+2}}{e^2}$

$\frac{dx}{dt} = \frac{\sqrt{4}}{e^2}$

$\frac{dx}{dt} = \frac{2}{e^2} > 0$

$\frac{dy}{dx}$ = slope

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$\frac{dy}{dx} = \frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}}$

@ $t=2$,

$\frac{dy}{dx} = \frac{\sin^2(2)}{\left(\frac{\sqrt{2+2}}{e^2}\right)}$

$\frac{dy}{dx} = \frac{e^2 \sin^2(2)}{\sqrt{4}}$

$\frac{dy}{dx} = \frac{e^2 \sin^2(2)}{2}$

$\frac{dy}{dx} = 3.055$

therefore, the horizontal movement of the particle is to the right at $t=2$ because $\frac{dx}{dt}$ is positive at $t=2$.

∴ the slope of the path of the particle is 3.055 at $t=2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

$x(t) = 1 + \int_2^t \frac{dx}{dT}\, dT$

$x(t) = 1 + \int_2^t \frac{\sqrt{T+2}}{e^T}\, dT$

$x(4) = 1 + \int_2^4 \frac{\sqrt{T+2}}{e^T}\, dT$

$x(4) = 1.253$

∴ The x-coordinate of the particle's position at time $t=4$ is 1.253

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(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

speed $= \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$

$= \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2+(\sin^2 t)^2}$

speed $= \sqrt{\frac{(t+2)}{e^{2t}}+\sin^4(t)}$

@ $t=4$,

speed $= \sqrt{\left(\frac{4+2}{e^{2(4)}}\right)+\sin^4(4)}$

speed $= .575$

∴ The speed of the particle at time $t=4$ was .575

$v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

$v(t) = \langle \frac{\sqrt{t+2}}{e^t}, \sin^2 t \rangle$

$a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

$a(t) = \langle \frac{(e^t)\frac{1}{2\sqrt{t+2}} - e^t\sqrt{t+2}}{e^{2t}}, 2\sin(t)\cos(t) \rangle$

$a(t) = \langle \frac{\frac{1}{2\sqrt{t+2}} - \sqrt{t+2}}{e^t}, 2\sin(t)(\cos(t)) \rangle$

$a(4) = \langle \frac{\frac{1}{2\sqrt{4+2}} - \sqrt{4+2}}{e^4}, 2\sin(4)\cos(4) \rangle$

$a(4) = \langle -.041, .989 \rangle$

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

distance $= \int_2^4 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$= \int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2+(\sin^2 t)^2}\,dt$

distance $= .651$

∴ The distance traveled by the particle from time $t=2$ to $t=4$ was .651

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2B1

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2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

   (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$$\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t} \Rightarrow x'(2) = \frac{\sqrt{2+2}}{e^2} = \frac{2}{e^2} \approx 0.271$$

To the right b/c $\frac{dx}{dt}$ [which represents (horizontal) velocity in the $\pm x$ direction] is positive.

$$\text{slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)} \Rightarrow @\ t=2 = \frac{\sin^2(2)}{\left(\frac{\sqrt{2+2}}{e^2}\right)} \approx 3.0547 \approx \boxed{3.055}$$

   (b) Find the x-coordinate of the particle's position at time $t = 4$.

$$x(t) = x(2) + \int_2^4 \frac{dx}{dt} = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt \approx 1 + 0.252954410\,8$$

$$x(4) \approx \boxed{1.253}$$

Continue problem 2 on page 7.

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(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{\sqrt{4+2}}{e^4}\right)^2 + \left[\sin^2(4)\right]^2} = 0.5745044453$$

$$\approx \boxed{0.575}$$

$$a(4) = \frac{d^2y/dt^2}{d^2x/dt^2} \approx \frac{2\sin 4 \cos^3 4}{\left(\frac{\frac{1}{2}(4+2)^{-1/2}(e^4)(e^4)(\sqrt{4+2})}{e^{2(4)}}\right)} :$$

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

$$\text{dist} = \int_2^4 \left|\frac{dy}{dx}\right| = \int_2^4 \left|\frac{\sin^2 t}{\left(\frac{\sqrt{t+2}}{e^t}\right)}\right| \approx 5.233420614 \approx \boxed{5.233}$$

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2C 1

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}} \overset{t=2}{=} \frac{\sin^2(2)}{\frac{\sqrt{2+2}}{e^2}} = 3.055$$

slope of the particle @ t=2.

The horizontal movement of the particle at t=2 is moving to the right b/c the slope is positive and t≥0.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

$$\int \frac{dx}{dt}$$

$$\int \frac{\sqrt{t+2}}{e^t}\,dt$$

$$\int \frac{(t+2)^{1/2}}{e^t}\,d$$

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Continue problem 2 on page 7.

2C2

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

$$speed = \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$$

$$= \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + \left(\sin^2 t\right)^2}$$

$$= \sqrt{\left(\frac{2}{e^2}\right)^2 + \left(\sin^2(2)\right)^2}$$

$$= \boxed{.7569}$$

$$acceler = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$$

$$= \sqrt{}$$

$y = \sin(t)^2$

$2(\sin(t)) \cdot \cos t$

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

$$TDT = \int_{t_1}^{t_2} \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}\, dt$$

$$TDT = \int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + \left(\sin^2 t\right)^2}\, dt$$

$$\boxed{TDT = .9004}$$

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AP® CALCULUS BC
2012 SCORING COMMENTARY

Question 2

Overview

This problem described the path of a particle whose position at time t is given by $(x(t), y(t))$, where $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$. Part (a) asked whether the particle's horizontal direction of motion is toward the left or toward the right at time $t = 2$. Students should have determined the sign of $\frac{dx}{dt}$ at this time to establish the direction of motion. Students were asked to find the slope of the particle's path at that time. The slope can be found by evaluating $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ at $t = 2$. Part (b) asked students to find the x-coordinate of the particle's position at time $t = 4$. This is calculated using the expression $x(4) = x(0) + \int_0^4 x'(t)\,dt$. Part (c) asked for the speed of the particle at time $t = 4$ seconds. This value is found by evaluating $\sqrt{(x'(t))^2 + (y'(t))^2}$ at time $t = 4$. Students were then asked for the acceleration vector at this time, which is given by $\langle x''(4), y''(4)\rangle$. Part (d) asked for the distance traveled by the particle over the interval $2 \le t \le 4$ seconds. This is found by integrating $\sqrt{(x'(t))^2 + (y'(t))^2}$ over the interval $2 \le t \le 4$.

Sample: 2A
Score: 9

The student earned all 9 points.

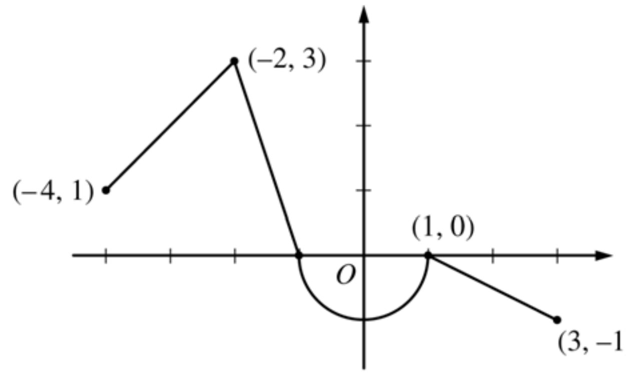
Sample: 2B
Score: 6

The student earned 6 points: 3 points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student correctly evaluates the speed. The expression for acceleration is incorrect. In part (d) the student presents an incorrect integral for distance.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student considers $\frac{dy/dt}{dx/dt}$ and calculates the slope correctly. The student's reason for the horizontal movement of the particle is incorrect. In parts (b) and (c) the student's work is not sufficient to earn any points. In part (d) the student's integral is correct, so 1 point was earned.

Question 3



Graph of f

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

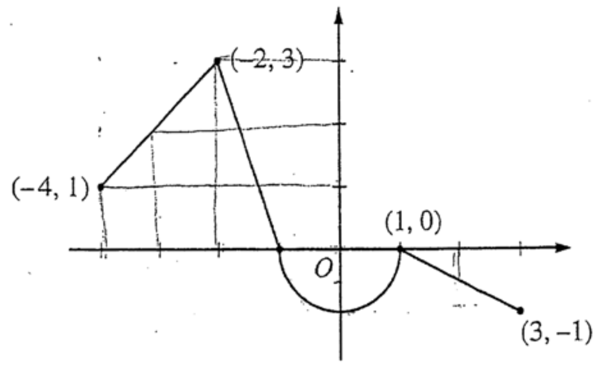
(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$ $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$ $= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$	$2: \begin{cases} 1: g(2) \\ 1: g(-2) \end{cases}$
(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$ $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$	$2: \begin{cases} 1: g'(-3) \\ 1: g''(-3) \end{cases}$
(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$. $g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$. $g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.	$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: x = -1 \text{ and } x = 1 \\ 1: \text{answers with justifications} \end{cases}$
(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

$g(2) = \int_1^2 f(t)\,dt$

$g(2) = -\frac{1}{2}(1)(\frac{1}{2})$

$\boxed{g(2) = -\frac{1}{4}}$

$g(-2) = \int_1^{-2} f(t)\,dt$

$g(-2) = \frac{1}{2}\pi(1)^2 - \left(\frac{1}{2}(1)(3)\right)$

$\boxed{g(-2) = \frac{\pi - 3}{2}}$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(x) = f(x)$

$\boxed{g'(-3) = 2}$

$g''(x) = f'(x)$

$\boxed{g''(-3) = 1}$

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Continue problem 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$g'(x) = f(x) = 0$

$x = -1, 1$

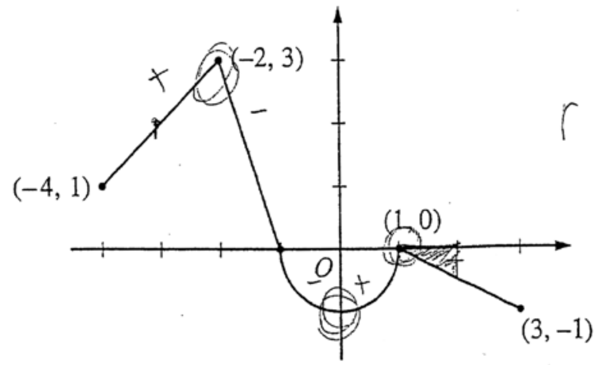
At $x=-1$ g has a relative maximum because $g'(x) = f(x)$ changes from positive to negative

At $x=1$ g has neither because $g'(x)=f(x)$ does not change sign

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

g has inflection points where $g''(x) = f'(x)$ changes sign. This occurs at $x = -2, 0, 1$

NO CALCULATOR ALLOWED



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

$A = \frac{bh}{2}$

$A = \frac{1(-\frac{1}{2})}{2}$

$A = -\frac{1}{4}$

$g(x) = \int_1^x f(t)\,dt$

$g(2) = \int_1^2 f(t)\,dt$

$g(2) = \frac{1(-.5)}{2}$

$\boxed{g(2) = -1/4}$

$g(-2) = -\int_{-2}^1 f(t)\,dt$

$A = \frac{bh}{2}$ $A = \pi r^2$

$-\left(\frac{1(3)}{2} - \frac{1}{2}\pi\right)$

$-3/2 + \frac{1}{2}\pi$

$\boxed{g(-2) = \frac{1}{2}\pi - 3/2}$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$m = \frac{\Delta y}{\Delta x}$

$-\frac{2}{2} = -1$

$(-3, 3)$

$g'(x) = f(x)$

$g'(-3) = f(-3)$

$\boxed{g'(-3) = 3}$

$g''(x) = f'(x)$

$g''(-3) = f'(-3)$

$\boxed{g''(-3) = -1}$

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Continue problem 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$g'(x) = 0 \leftarrow$ horizontal tangent line

$g'(x) = f(x) = 0$

at $x = -1$, $x = 1$

g'	+		−		−
g	inc	−1	dec	1	dec

at $x=-1$, there is a rel. max.
f' changes from + to −

at $x=1$, there is neither a maximum nor a minimum because the slope remains negative

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$g''(x) = f'(x)$

g will have a POI where the slope of f changes signs

at $\boxed{x=-2,\ x=0,\ x=1}$

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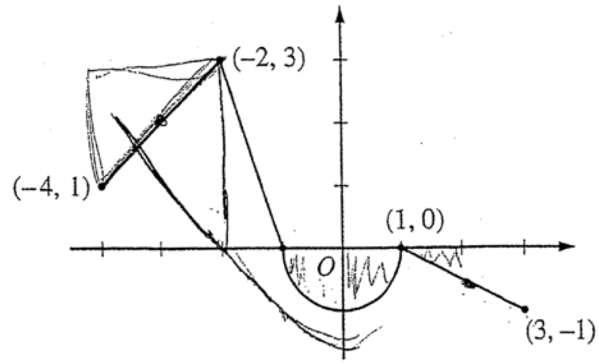
NO CALCULATOR ALLOWED

3C1



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

$g'(x) = f(x)$

$g(2) = \pi(1)^2 \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4}\pi + \frac{1}{4}$

$g(-2) = \frac{-1}{4}(1)^2\pi + \frac{1}{2} \cdot 3 \cdot 1 = -\frac{1}{4}\pi + \frac{3}{2}$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(x) = f(x)$

$g'(-3) = f(3) = 2$

$g''(x) = f'(x)$

$g''(-3) = f'(-3) = 1$

Do not write beyond this border.

Continue problem 3 on page 15.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$g'(x)=0 \quad g'(x)=f(x)=0$

$g'(x)=f(x)=0$ at $x=-1$

The point at $x=-1$ is a maximum because the graph of $f(x)|g'(x)$ transitions from positive to negative at this point. This means that on the original graph $(g(x))$ the graph is moving from increasing to decreasing at this point, which is representative of a maximum.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$g''(x)=f'(x)$

There is a point of inflection at $x=-2,3$ because on the $f(x)|g'(x)$ graph they are presented as extrema. Extrema on a first derivative graph represents a point of inflection on the original graph. Therefore the extrema on the graph given, $x=-2,3$, are points of inflection on $g(x)$.

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Question 3

Overview

This problem described a function f that is defined and continuous on the interval $[-4, 3]$. The graph of f on $[-4, 3]$ is given and consists of three line segments and a semicircle. The function g is defined by $g(x) = \int_1^x f(t)\,dt$. Part (a) asked for the values of $g(2)$ and $g(-2)$. These values are given by $\int_1^2 f(t)\,dt$ and $\int_1^{-2} f(t)\,dt$, respectively, and are computed using geometry and a property of definite integrals. Part (b) asked for the values of $g'(-3)$ and $g''(-3)$, provided they exist. Students should have applied the Fundamental Theorem of Calculus to determine that $g'(-3) = f(-3)$ and $g''(-3) = f'(-3)$. Students should have used the graph provided to determine the value of f and the slope of f at the point where $x = -3$. Part (c) asked for the x-coordinate of each point where the graph of g has a horizontal tangent line. Students were then asked to classify each of these points as the location of a relative minimum, relative maximum, or neither, with justification. Students should have recognized that horizontal tangent lines for g occur where the derivative of g takes on the value 0. These values can be read from the graph. Students should have applied a sign analysis to f in order to classify these critical points. Part (d) asked for the x-coordinates of points of inflection for the graph of g on the interval $-4 < x < 3$. Students should have reasoned graphically that these occur where f changes from increasing to decreasing, or vice versa.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In parts (a) and (d) the student's work is correct. In part (b) the student does not supply the correct values. In part (c) the student correctly considers $g'(x) = 0$ and identifies the correct x-values. The student does not give a correct justification for $x = -1$, confusing f' with g', and does not specify which function's slope is intended in the justification for $x = 1$.

Sample: 3C
Score: 3

The student earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not supply the correct values. In part (b) the student's work is correct. In part (c) the student considers $g'(x) = 0$, so the first point was earned. The student identifies only one of the x-values, so the second point was not earned. The student is not eligible for the third point. In part (d) the student does not identify the correct x-values.

Question 4

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

(a) $f(1) = 15,\quad f'(1) = 8$

An equation for the tangent line is
$y = 15 + 8(x - 1)$.

$f(1.4) \approx 15 + 8(1.4 - 1) = 18.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\int_1^{1.4} f'(x)\,dx \approx (0.2)(10) + (0.2)(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$

$f(1.4) \approx 15 + 4.6 = 19.6$

$3 : \begin{cases} 1 : \text{midpoint Riemann sum} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $f(1.2) \approx f(1) + (0.2)(8) = 16.6$

$f(1.4) \approx 16.6 + (0.2)(12) = 19.0$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(d) $T_2(x) = 15 + 8(x - 1) + \frac{20}{2!}(x - 1)^2$
$\quad = 15 + 8(x - 1) + 10(x - 1)^2$

$f(1.4) \approx 15 + 8(1.4 - 1) + 10(1.4 - 1)^2 = 19.8$

$2 : \begin{cases} 1 : \text{Taylor polynomial} \\ 1 : \text{approximation} \end{cases}$

NO CALCULATOR ALLOWED

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y - 15 = 8(x-1)$

$\boxed{y = 8x + 7}$

$y(1.4) \approx 8(1.4) + 7$

$= 11.2 + 7$

$= \boxed{18.2}$

1.4
8
11.2

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$\int_1^{1.4} f'(x)\,dx \approx (1.2-1)(10) + (1.4-1.2)(13)$

$= (0.2)(10) + (0.2)(13)$

$= 2 + 2.6 = \boxed{4.6}$

$f(1.4) \approx f(1) + \int_1^{1.4} f'(x)\,dx$

$= 15 + 4.6$

$= \boxed{19.6}$

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Continue problem 4 on page 17.

4A 2

NO CALCULATOR ALLOWED

16.6
2.4
19.0

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$f(1.2) \approx f(1) + f'(1)(0.2) = 15 + (8)(0.2) = 15 + 1.6 = 16.6$

$f(1.4) \approx f(1.2) + f'(1.2)(0.2) = 16.6 + (12)(0.2) = 16.6 + 2.4$

$= 19$

$\therefore \boxed{f(1.4) \approx 19}$

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(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$P_2(x) = \dfrac{f(1)(x-1)^0}{0!} + \dfrac{f'(1)(x-1)^1}{1!} + \dfrac{f''(1)(x-1)^2}{2!}$

$P_2(x) = 15 + \dfrac{8(x-1)}{1!} + \dfrac{20(x-1)^2}{2!}$

$\boxed{P_2(x) = 15 + 8(x-1) + 10(x-1)^2}$

$f(1.4) \approx 15 + 8(1.4-1) + 10(1.4-1)^2$

$= 15 + 8(0.4) + 10(0.4)^2$

$= 15 + 3.2 + 1.6$

$= \boxed{19.8}$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y = mx + b$

$m = 8$

$15 = 8(1) + b$

$7 = b$

$f(x) = 8x + 7$

$f(x) = 8(1.4) + 7$

$= 14.2$

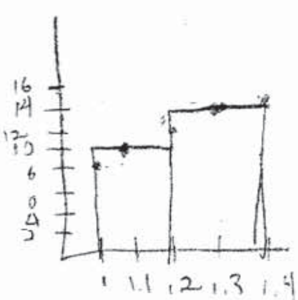
$1.4 \times 8 = 11.2$

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$[(1.2-1)(10) + (1.4-1.2)(13)]$

$[0.2(10) + 0.2(13)]$

$[2 + 2.6] \approx 4.6$



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Continue problem 4 on page 17.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$\Delta x = 0.2$

$f(1) = 15 \qquad (1, 15)$

$f^2 = 15 + 8(0.2) = 15 + 1.6 = 16.6 \quad (1.2, 16.6)$

$f(1.4) = 16.6 + 12(0.2) = 16.6 + 2.4 = 17.4 \quad (1.4, 17.4)$

$f(1.4) \approx 17.4$

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(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$\frac{15(x-1)^0}{0!} + \frac{8}{1!}(x-1)^1 + \frac{20}{2!}(x-1)^2$

$P_2(x) = 15 + 8(x-1) + 10(x-1)^2$

$= 15 + 8(1.4-1) + 10(1.4-1)^2$

$= 15 + 8(0.4) + 10(0.4)^2$

$= 15 + 3.2 + 10(.16)$

$= 18.2 + 1.60 \approx 19.80$

NO CALCULATOR ALLOWED

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

$y - y_1 = m(x - x_1)$ $\quad m = dy/dx = f'(x)$

$y - 15 = 8(x-1)$

$y = 8x + 14$

$y = (8)(1.4) + 14$

$1.4 \times 8 = 11.2$

$y = 11.2 + 14$

$y = 25.2$

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$\frac{1}{2}\int_1^{1.4} 10(2) + 13(2)$

$\frac{1}{2}\int 20 + 26$

$\frac{1}{2}(46)$

23

Continue problem 4 on page 17.

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 4C 2

NO CALCULATOR ALLOWED

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$\Delta y = m \cdot \Delta x$

$\Delta y = 8 \cdot .2$

$\Delta y = 1.6$

$y = 15 + 1.6 = 16.6$

$(1.2, 16.6)$

$\Delta y = 12 \cdot .2$

$\Delta y = 2.4$

$y = 16.6 + 2.4$

$(1.4, 19.0)$

$8 \times 2 = 1.6$

$16.6 + 2.4 = 19.0$

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$8 + 12(x-1)' + \frac{14.5(x-1)^2}{2!}$

$8 + 12(1.4-1) + \frac{14.5(1.4-1)^2}{2!}$

$8 + 4.8 + 1.45 =$

$f(1.4) = 14.25$

$.4^2$ $\quad .4 \times .4 = .16$ $\quad 12 \times .16 = 72,\ 120$

$6(.2)$

$12 \times .4 = 4.8$

$7.25 \times .2 = 1.450$

$8 + 4.8 + 1.45 = 14.25$

GO ON TO THE NEXT PAGE.

Question 4

Overview

Students were presented with a table of values for f' at selected values of x given that f is a twice-differentiable function. The values for $f(1)$ and $f''(1)$ are also given. Part (a) asked students to write an equation for the line tangent to the graph of f at $x = 1$ and then use this line to approximate $f(1.4)$. Students should have used the given values for $f(1)$ and $f'(1)$ to construct an equation equivalent to $y = f(1) + f'(1)(x - 1)$. Students could then substitute $x = 1.4$ to obtain the desired approximation. Part (b) asked students to use a midpoint Riemann sum with two subintervals of equal length, based on values in the table, to approximate $\int_1^{1.4} f'(x)\, dx$. They were then asked to use this approximation to estimate $f(1.4)$. This estimate is obtained by using the midpoint Riemann sum in the expression $f(1) + \int_1^{1.4} f'(x)\, dx$. Part (c) asked students to use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Part (d) asked for the second-degree Taylor polynomial for f about $x = 1$, which was then used to obtain yet another approximation for $f(1.4)$. Students should have used the given values for $f(1)$, $f'(1)$, and $f''(1)$ to write the Taylor polynomial.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In parts (a) and (d) the student's work is correct. In part (b) the student presents a correct midpoint Riemann sum, so the first point was earned. In part (c) the student correctly presents Euler's method with two steps, so the first point was earned. The student's arithmetic error in the last sum leads to an incorrect value for the approximation.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student presents the correct tangent line and earned the first point. The student makes an algebra error in subsequent work and did not earn the second point. In part (b) the student's work is incorrect. In part (c) the student's work is correct. In part (d) the student uses incorrect values for $f(1)$, $f'(1)$, and $f''(1)$.

Question 5

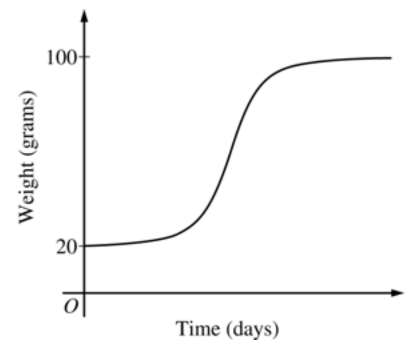
The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

NO CALCULATOR ALLOWED

5A₁

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

when it is 40 grams: $\frac{dB}{dt} = \frac{1}{5}(100-40) = 12$ g/day

when it is 70 grams: $\frac{dB}{dt} = \frac{1}{5}(100-70) = 6$ g/day

so the bird is gaining weight faster when it weighs 40 grams.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

$\frac{dB}{dt} = 20 - \frac{1}{5}B$

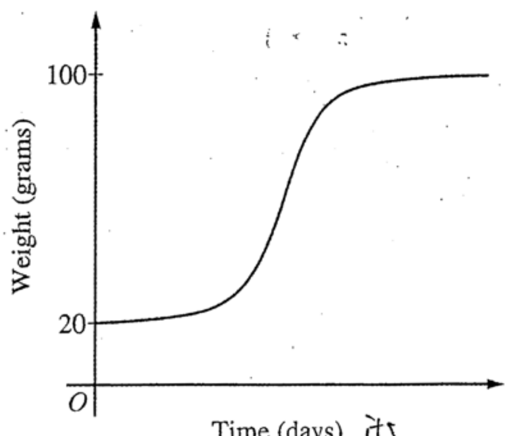
$\frac{d^2B}{dt^2} = -\frac{1}{5} \cdot \frac{dB}{dt}$

$= -\frac{1}{5}(20 - \frac{1}{5}B)$

$= \frac{1}{25}B - 4$

$\frac{1}{25}B - 4 > 0$

$B > 100$



so, the graph cannot be concave up when its weight is below 100 g

Continue problem 5 on page 19.

5A2

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(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\frac{1}{\frac{1}{5}(100-B)}\, dB = dt$$

$$\frac{5}{100-B}\, dB = dt$$

$$\int \frac{5}{100-B}\, dB = \int 1\, dt$$

$$-5\ln|100-B| = t + c$$

$$\ln(100-B) = -\frac{1}{5}(t+c)$$

$$100 - B = e^{-\frac{1}{5}(t+c)}$$

$$B = 100 - e^{-\frac{1}{5}(t+c)}$$

$$20 = 100 - e^{-\frac{1}{5}c}$$

$$e^{-\frac{1}{5}c} = 80$$

$$-\frac{1}{5}c = \ln 80$$

$$c = -5\ln 80$$

$$\therefore B = 100 - e^{-\frac{1}{5}(t - 5\ln 80)}$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

5B1

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

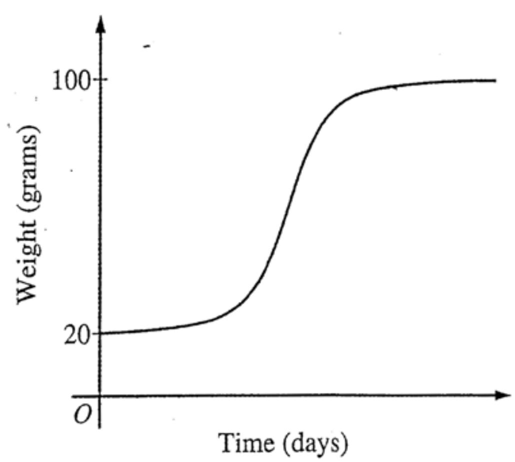
(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

Weight $= 40 \Rightarrow \frac{dB}{dt} = \frac{1}{5}(100-40) = \frac{60}{5}$ gram/day

Weight $= 70 \Rightarrow \frac{dB}{dt} = \frac{1}{5}(100-70) = \frac{30}{5}$ gram/day

$\frac{60}{5} > \frac{30}{5} \Rightarrow$ at weight $= 40$ gram, the rate of change of Bird weight is faster.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



$\frac{d^2B}{dt^2} = \frac{1}{5}\left(0 - \frac{dB}{dt}\right) = -\frac{1}{5}\frac{dB}{dt}$. , $\frac{d^2B}{dt^2}$ is negative

$\Rightarrow$ the graph of B has to be concave down all the times

Continue problem 5 on page 19.

NO CALCULATOR ALLOWED

5B2

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\int \frac{dB}{100-B} = \int \frac{1}{5}\, dt$$

$$\ln(100 - B) = \frac{t}{5} + C$$

$$100 - B = Ce^{t/5}$$

$$B(0) = 20 \Rightarrow 100 - 20 = Ce^{0} \Rightarrow C = 80$$

$\Rightarrow$ partialar solution: $\boxed{100 - B = 80e^{t/5}}$

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Do not write beyond this border.

5C1

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

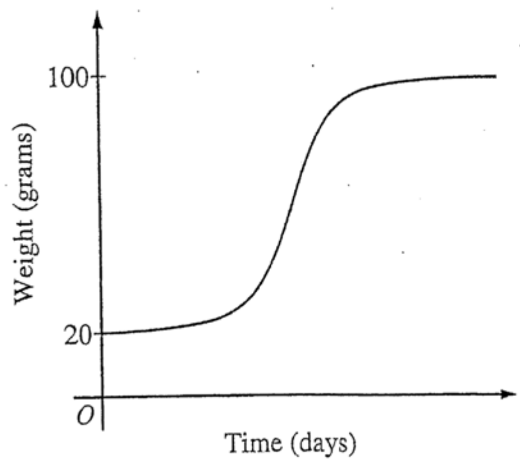
(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

$\frac{1}{5}(100-40) \quad \frac{60}{5} \quad 12$

$\frac{1}{5}(100-70) \quad \frac{30}{5} \quad 6$

gains weight faster when it weighs 40 grams because its growing at twice the rate it is when its 70 grams.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



$20 - \frac{B}{5}$

$\frac{d^2B}{dt^2} = -\frac{1}{5}$

Because $\frac{d^2B}{dt^2}$ is $-\frac{1}{5}$, this can't resemble the following because the concavity isn't negative.

Continue problem 5 on page 19.

5C2

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\int \frac{1}{5}\,dt = \int \frac{dB}{100-B}$$

$$C + \frac{1}{5}t = -\frac{1}{2}(100-B)^{-2}$$

$$\frac{1}{5}t + C = \frac{-1}{2(100-B)^2}$$

$$-2\left(\frac{1}{5}t + C\right) = \frac{1}{100-B^2}$$

$$100 - B^2 = \frac{1}{-2\left(\frac{1}{5}t + C\right)}$$

$$100 + \frac{1}{+2\left(\frac{1}{5}t + C\right)} = B^2$$

$$\sqrt{100 + \frac{1}{2\left(\frac{1}{5}t + C\right)}} = B$$

$$\sqrt{100 + \frac{1}{2(C)}} = 20$$

$$\begin{array}{r} 20 \\ 20 \\ \hline 400 \end{array}$$

$$\frac{1}{2C} = 300$$

$$\frac{\frac{1}{300} = 2C}{2}$$

$$C = \frac{2}{300}$$

$$B = \sqrt{100 + \frac{1}{2\left(\frac{1}{5}t + \frac{2}{300}\right)}}$$

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Question 5

Overview

The context of this problem is weight gain of a baby bird. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. A function B modeling the weight of the bird satisfies $\frac{dB}{dt} = \frac{1}{5}(100 - B)$, where t is measured in days since the bird was first weighed. Part (a) asked whether the bird is gaining weight faster when it weighs 40 grams or when it weighs 70 grams. Students had to evaluate and compare $\frac{dB}{dt}$ for these two values of B. Part (b) asked for $\frac{d^2B}{dt^2}$ in terms of B. Students should have used a sign analysis of the second derivative to explain why the graph of B cannot resemble the given graph. Part (c) asked students to use separation of variables to solve the initial value problem $\frac{dB}{dt} = \frac{1}{5}(100 - B)$ with $B(0) = 20$ to find $B(t)$.

Sample: 5A
Score: 9

The student earned all 9 points. Note that in part (c) the student does not need absolute value on the fifth line because $B(0) = 20$.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In part (a) the student's work is correct. In part (b) the first point was not earned because the student does not present $\frac{d^2B}{dt^2}$ in terms of B. The student's correct appeal to the chain rule and correct explanation earned the second point. In part (c) the student earned the first point with a correct separation on the second line. The second point was not earned because the student's antiderivative on the left-hand side on the third line is incorrect. (The antiderivative should be $-\ln(100 - B)$, with no absolute value needed.) A student who did not earn the second point is not eligible for the fifth point. The student earned the third point on the third line and the fourth point on the fifth line for correctly substituting 0 for t and 20 for B.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). In part (a) the student's work is correct. In part (b) the student makes a chain rule error and did not earn the first point. The student is not eligible for the second point in part (b). In part (c) the student presents a correct separation on the first line and earned the first point. The student's incorrect B-antiderivative makes the student ineligible for any additional points in part (c).

Question 6

The function g has derivatives of all orders, and the Maclaurin series for g is
$\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

(a) $\left|\dfrac{x^{2n+3}}{2n+5} \cdot \dfrac{2n+3}{x^{2n+1}}\right| = \left(\dfrac{2n+3}{2n+5}\right) \cdot x^2$

$\displaystyle\lim_{n\to\infty}\left(\frac{2n+3}{2n+5}\right) \cdot x^2 = x^2$

$x^2 < 1 \;\Rightarrow\; -1 < x < 1$
The series converges when $-1 < x < 1$.

When $x = -1$, the series is $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$
This series converges by the Alternating Series Test.

When $x = 1$, the series is $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \cdots$
This series converges by the Alternating Series Test.

Therefore, the interval of convergence is $-1 \le x \le 1$.

$5: \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies interior of interval of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

(b) $\left|g\left(\frac{1}{2}\right) - \frac{17}{120}\right| < \dfrac{\left(\frac{1}{2}\right)^5}{7} = \dfrac{1}{224} < \dfrac{1}{200}$

$2: \begin{cases} 1 : \text{uses the third term as an error bound} \\ 1 : \text{error bound} \end{cases}$

(c) $g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \cdots + (-1)^n\left(\frac{2n+1}{2n+3}\right)x^{2n} + \cdots$

$2: \begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$

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6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2(n+1)+1}}{2(n+1)+3}\cdot\frac{2n+3}{(-1)^n x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2n+2+1}}{2n+2+3}\cdot\frac{2n+3}{(-1)^n x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(-1)^{\cancel{n+1}}x^{2n+3}}{\cancel{2n+5}}\cdot\frac{\cancel{2n+3}}{\cancel{(-1)}^n x^{2n+1}}\right|$$

$$= |x^2| < 1$$

$$-1 < x < 1$$

end point

$x=-1$ $\quad (-1)^n\cdot\frac{(-1)^{2n+1}}{2n+3} = \frac{(-1)^{3n+1}}{2n+3}$

~~the~~ series is alternating, and the absolute value of each ~~t~~term decreases to 0

$\therefore$ converges

$x=1$ $\quad (-1)^n\cdot\frac{1^{2n+1}}{2n+3} = \frac{(-1)^n}{2n+3}$

the series is alternating, and the absolute value of each term decreases to 0 $\therefore$ converges

$\therefore$ interval of convergence is $x \in [-1, 1]$.

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Continue problem 6 on page 21.

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(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

$\because$ The Maclaurin series for g evaluated at $x=\frac{1}{2}$ is an alternating series whose decrease in absolute value to 0

$\therefore$ Error of using the first two nozero terms is smaller than the third term of the maclaurin series

third term: $x=\frac{1}{2}$

$$\frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{32}\cdot\frac{1}{7} = \frac{1}{224} < \frac{1}{200}$$

$\therefore$ the approximation differs from $g(\frac{1}{2})$ is less than $\frac{1}{200}$

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

$$g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4 + \cdots + (-1)^n \cdot (2n+1)\frac{x^{2n}}{2n+3}$$

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6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{2(n+1)+3} \cdot \frac{2n+3}{x^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{x^2 \cdot 2n+3}{2n+5} \right| = |x^2| < 1$$

$$-1 < x < 1$$

when $x = 1$, $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+3} = \frac{1}{3} - \frac{1}{5} + \frac{1}{7} \cdots$ Convergent

when $x = -1$, $\sum_{n=0}^{\infty} (-1)^n \frac{(-1)}{2n+3} = -\frac{1}{3} + \frac{1}{5} - \frac{1}{7}$. Convergent

So the interval of convergence of Maclaurin series for g is $-1 \le x \le 1$.

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Continue problem 6 on page 21.

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(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

Suppox $a_n = (-1)^n \frac{x^{2n+1}}{2n+3}$, $x = \frac{1}{2}$

$a_n = (-1)^n \frac{(\frac{1}{2})^{2n+1}}{2n+3}$ $g(\frac{1}{2})$ is gained using the first two terms,

So difference

$< |a_3| = \frac{(\frac{1}{2})^7}{9} = \frac{1}{2^7 \times 9} < \frac{1}{200}$

So this approximation differs from $g(\frac{1}{2})$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

Maclaurin series for $g'(x)$

$g'(x) = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7} + \cdots\cdots \frac{(-1)^n(2n+1)x^{2n}}{2n+3} + \cdots$

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6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

$2(n+1)+1$ $\quad$ $2(n+1)+3$

$2n+2+1$ $\quad$ $2n+2+3$

(a) Using the ratio test, determine the interval of convergence of $\boxed{\text{the Maclaurin}}$ series for g.

$$\lim_{n\to\infty}\left|\frac{x^{2n+3}}{2n+5}\cdot\frac{2n+3}{x^{2n+1}}\right| < 1$$

$$\lim_{n\to\infty}\frac{x^3+3}{x+5} < 1$$

$$x^3+3 < x+5$$

$$x^3 - x < 2$$

$$x(x^2-1) < 2$$

$$x(x-1)(x+1) < 2$$

$x=0 \quad x=1 \quad x=-1$

$$-1 \le x \le 1$$

$$\frac{(-1)^3+3}{-1+5} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{(1)^3+3}{1+5} = \frac{4}{6} = \frac{2}{3}$$

Continue problem 6 on page 21.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(Error)

$$g\left(\tfrac{1}{2}\right) = \frac{17}{120}$$

$$\begin{array}{r} 1 \\ 32 \\ \times 7 \\ \hline 224 \end{array}$$

$$\frac{x}{3} - \frac{x^3}{5} = \frac{17}{120}$$

Proving $\frac{x^5}{7} = \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{\frac{1}{32}}{7} = \frac{1}{224}$

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

Maclaurin: $g(x) = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7}$

take derivative

$$g'(x) = \frac{1}{3} - \frac{1}{5}(3x^2) + \frac{1}{7}(5x^4)$$

$$g'(x) = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7}$$

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Question 6

Overview

This problem presented the Maclaurin series for an infinitely differentiable function g. Part (a) asked students to use the ratio test to determine the interval of convergence for the given Maclaurin series. Students should have observed that for $x = -1$ and $x = 1,$ the resulting series is alternating with terms decreasing in absolute value to 0. Therefore, the series converges for $x = -1$ and $x = 1.$ Part (b) asked students to show that the approximation for $g\left(\frac{1}{2}\right)$ obtained by using the first two nonzero terms of the series differs from the actual value by less than $\frac{1}{200}$. Because this is an alternating series with terms decreasing in absolute value to 0, students should have observed that the absolute value of the third term bounds the error and is strictly less than $\frac{1}{200}$. Part (c) asked the students to find the first three nonzero terms and the general term of the Maclaurin series for $g'(x).$ Students should have computed the symbolic derivative of the first three nonzero terms and the general term of the series for $g(x).$

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 4 points in part (a), no points in part (b), and 2 points in part (c). In part (a) the student sets up the ratio correctly, evaluates the limit, finds the interior of the interval of convergence, and considers the endpoints. The student does not provide a reason for the convergence, so the fifth point in part (a) was not earned. In part (b) the student does not use the third term as the error bound for the first two terms, so no points were earned. In part (c) the student's work is correct.

Sample: 6C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student sets up the ratio correctly, so the first point was earned. In part (b) the student selects the third term as the error bound for the sum of the first two terms, evaluates the third term, but never states that the error is less than $\frac{1}{200}$. In part (c) the student correctly finds the first three terms but not the general term.